Modeling Data Using Linear Functions

A.1(A) Describe independent and dependent quantities in functional relationships;

A.1(B) Gather and record data and use data sets to determine functional relationships between quantities;

A.1(C) Describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;

A.1(D) Represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

A.1(E) Interpret and make decisions, predictions, and critical judgments from functional relationships.

A.2(B) Identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete.

A.2(D) Collect and organize data, make and interpret scatterplots (including recognizing positive, negative, or no correlation for data approximating linear situations), and model, predict, and make decisions and critical judgments in problem situations.

A.5(C) Use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

Materials

Advanced Preparation:

- Student access to computers with Internet access and/or a projection device to use to gather data as a class.

For each student:

- Graphing calculator
- Computer with Internet access
- Optional software: Microsoft Excel or TI-Interactive
- **Road Construction** activity sheet
- **The Price of Concrete in Oregon** activity sheet
- **Place Your Bids** activity sheet
- **City Folk, Country Folk** activity sheet

For each student group of 3 - 4 students:

- Chart Paper
- Meter sticks
- Markers

ENGAGE

The Engage portion of the lesson is designed to generate student interest in the factors that influence transportation costs and road construction. This part of the lesson is designed for groups of three to four students.

1. Distribute the **Road Construction** activity sheet. Students should respond to the prompts on the sheet.
2. After each group has had time to respond to the three prompts, ask the student groups to share their responses with the whole class.
3. Have a student volunteer record the results on the overhead, on a transparency or on the chalkboard.
4. Debrief the activity using the Facilitation Questions.

Facilitation Questions – Engage Phase

- What were the most common responses?
  Answers may vary. Student groups may have recorded costs of materials and costs of labor.
- How can we determine what some of these costs are?
  Answers may vary. Students may suggest Internet research or contacting a local construction company.
- Who is "in charge" of roads and highways?
  Answers may vary. Students should be aware that some roads are managed by county, state, or federal authorities.

Have the students get on the internet and go the website: http://www.fhwa.dot.gov.programadmin/pt2004q1.htm. The students must scroll down in order to see the data in the tables. Discuss with the students the meaning of the different aspects of road construction: excavation, surfacing and structures. Although several data charts are available, students will be using the first chart which displays price trends for federal aid highway construction.

EXPLORE

The Explore portion of the lesson provides the student with an opportunity to be actively involved in the exploration of the mathematical concepts addressed. This part of the lesson is designed for groups of three to four students.

1. Distribute the **The Price of Concrete in Oregon** activity sheet.
2. Have students follow the directions on the activity sheet to collect data and explore the cost of Portland cement concrete over time from 1972 until 2000.
   *Note: The time interval between data points changes in 2001 from annually to quarterly. If students are going to use data points after 2000, they need to only use the **annual** averages for subsequent years.*

   Students may use the graphing calculator, Excel or TI-Interactive for this portion of the lesson. If students need extra help with using Excel or TI-Interactive, provide them with access to "Technology Tutorial: Road Construction."

Facilitation Questions – Explore Phase

Data Exploration:

- Which relationships appear to be linear?
  Answers may vary. One possible relationship is the Average contract price (sq. yd.) of Portland cement concrete versus the year number.
- Why do you think the relationship is linear?
  Answers may vary. Students may calculate rates of change.

Data Collection:

- Which relationships show a positive correlation?
  Answers may vary. Students should be able to justify their selections.
- Which relationships show a negative correlation?
  Answers may vary. Students should be able to justify their selections.
- Which relationships show no correlation?
  Answers may vary. Students should be able to justify their selections.

Data Analysis:

- How can you determine if data can be modeled by a linear function?
  Answers may vary. Participants may calculate rates of change.
- What is the parent function for linear functions?
  $y = x$
- How can you determine the values of the parameters for a linear function?
  Answers may vary. Possibilities include using transformations, and finding rates of change.
- What other method can you use to determine a function rule?
  Participants should realize there are several methods that may be used to determine the function rule.

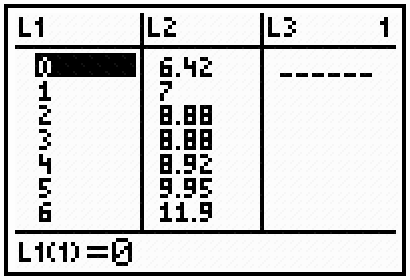
EXPLAIN

The Explain portion of the lesson is directed by the teacher to allow the students to formalize their understanding of the TEKS addressed in the lesson. In this phase, debrief the ***The Price of Concrete in Oregon*** *activity sheet from the Explore. Use the Facilitation Questions to prompt student groups to share their responses to the data analysis.*

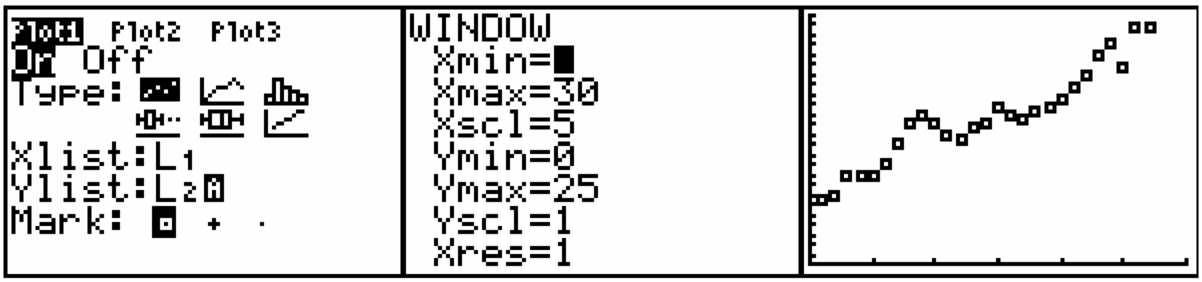
1. Debrief the **The Price of Concrete in Oregon** activity sheet.

2. Ask students to share how they developed their scatterplots and why they chose this method.

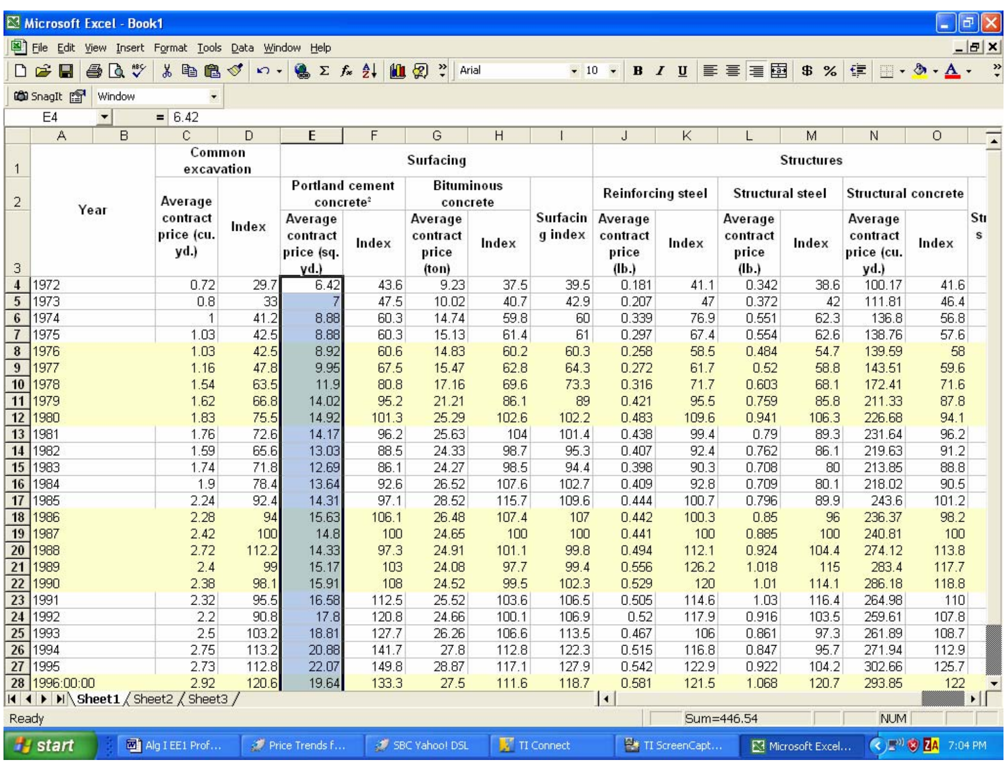
Sample answer using a Graphing calculator:
Enter data into the STAT lists.



Set up the STAT PLOT, choose an appropriate window, and view the graph.

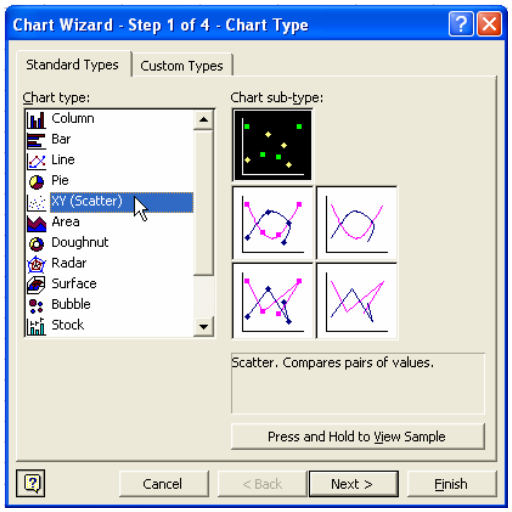


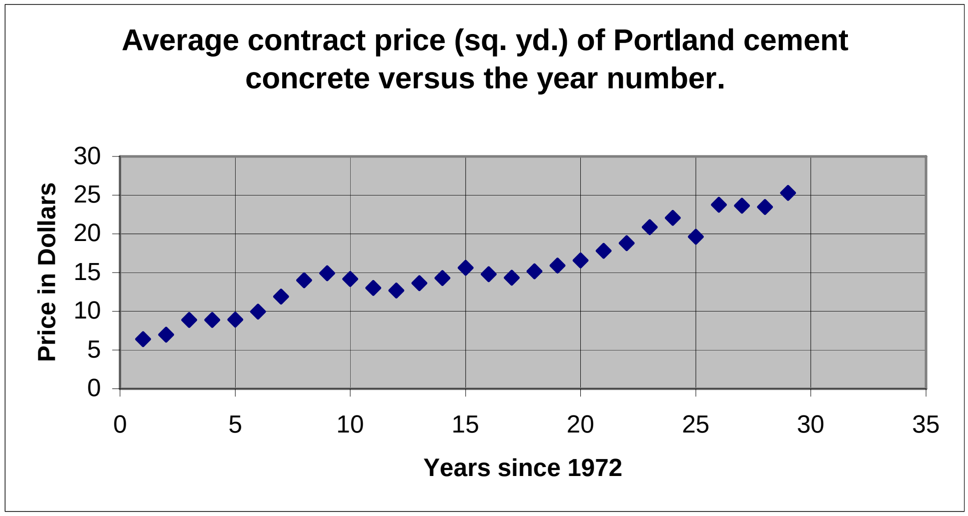
Sample answer using a Spreadsheet:
Copy and Paste the data into a blank spreadsheet. Select the data you are investigating.



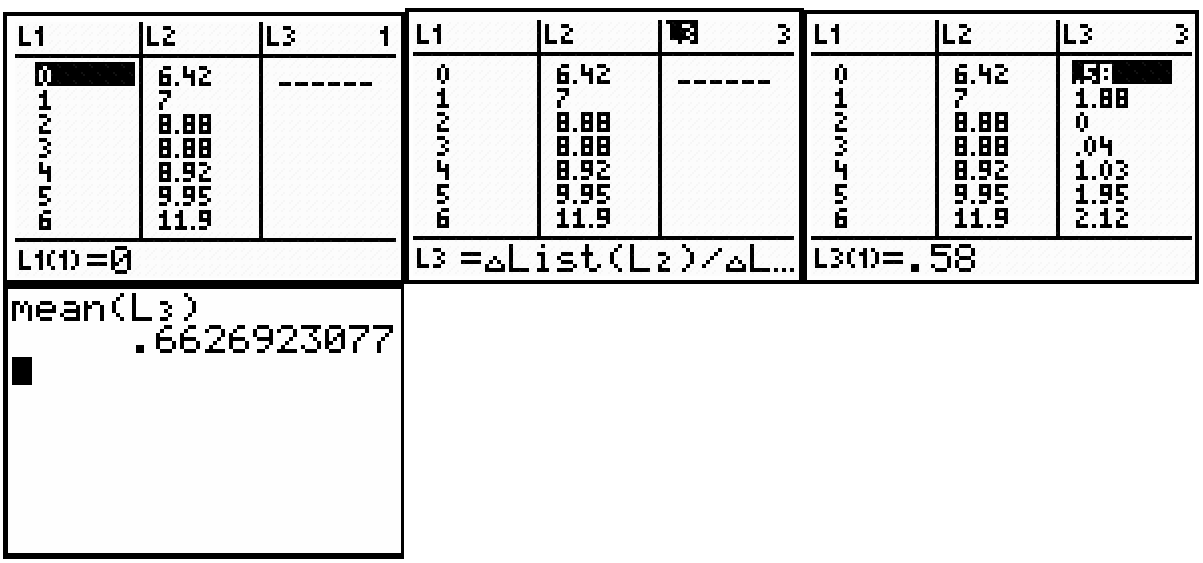
	A	C	D	E	F	G	H	I	J	K	L	M	N	O
1		Common excavation		Surfacing					Structures					
2	Year			Portland cement concrete²		Bituminous concrete			Reinforcing steel		Structural steel		Structural concrete	
3		Average contract price (cu. yd.)	Index	Average contract price (sq. yd.)	Index	Average contract price (ton)	Index	Surfacing index	Average contract price (lb.)	Index	Average contract price (lb.)	Index	Average contract price (cu. yd.)	Index
4	1972	0.72	29.7	6.42	43.6	9.23	37.5	39.5	0.181	41.1	0.342	38.6	100.17	41.6
5	1973	0.8	33	7	47.5	10.02	40.7	42.9	0.207	47	0.372	42	111.81	46.4
6	1974	1	41.2	8.88	60.3	14.74	59.8	60	0.339	76.9	0.551	62.3	136.8	56.8
7	1975	1.03	42.5	8.88	60.3	15.13	61.4	61	0.297	67.4	0.554	62.6	138.76	57.6
8	1976	1.03	42.5	8.92	60.6	14.83	60.2	60.3	0.258	58.5	0.484	54.7	139.59	58
9	1977	1.16	47.8	9.95	67.5	15.47	62.8	64.3	0.272	61.7	0.52	58.8	143.51	59.6
10	1978	1.54	63.5	11.9	80.8	17.16	69.6	73.3	0.316	71.7	0.603	68.1	172.41	71.6
11	1979	1.62	66.8	14.02	95.2	21.21	86.1	89	0.421	95.5	0.759	85.8	211.33	87.8
12	1980	1.83	75.5	14.92	101.3	25.29	102.6	102.2	0.483	109.6	0.941	106.3	226.68	94.1
13	1981	1.76	72.6	14.17	96.2	25.63	104	101.4	0.438	99.4	0.79	89.3	231.64	96.2
14	1982	1.59	65.6	13.03	88.5	24.33	98.7	95.3	0.407	92.4	0.762	86.1	219.63	91.2
15	1983	1.74	71.8	12.69	86.1	24.27	98.5	94.4	0.398	90.3	0.708	80	213.85	88.8
16	1984	1.9	78.4	13.64	92.6	26.52	107.6	102.7	0.409	92.8	0.709	80.1	218.02	90.5
17	1985	2.24	92.4	14.31	97.1	28.52	115.7	109.6	0.444	100.7	0.796	89.9	243.6	101.2
18	1986	2.28	94	15.63	106.1	26.48	107.4	107	0.442	100.3	0.85	96	236.37	98.2
19	1987	2.42	100	14.8	100	24.65	100	100	0.441	100	0.885	100	240.81	100
20	1988	2.72	112.2	14.33	97.3	24.91	101.1	99.8	0.494	112.1	0.924	104.4	274.12	113.8
21	1989	2.4	99	15.17	103	24.08	97.7	99.4	0.556	126.2	1.018	115	283.4	117.7
22	1990	2.38	98.1	15.91	108	24.52	99.5	102.3	0.529	120	1.01	114.1	286.18	118.8
23	1991	2.32	95.5	16.58	112.5	25.52	103.6	106.5	0.505	114.6	1.03	116.4	264.98	110
24	1992	2.2	90.8	17.8	120.8	24.66	100.1	106.9	0.52	117.9	0.916	103.5	259.61	107.8
25	1993	2.5	103.2	18.81	127.7	26.26	106.6	113.5	0.467	106	0.861	97.3	261.89	108.7
26	1994	2.75	113.2	20.88	141.7	27.8	112.8	122.3	0.515	116.8	0.847	95.7	271.94	112.9
27	1995	2.73	112.8	22.07	149.8	28.87	117.1	127.9	0.542	122.9	0.922	104.2	302.66	125.7
28	1996:00:00	2.92	120.6	19.64	133.3	27.5	111.6	118.7	0.581	121.5	1.068	120.7	293.85	122

Create an XY-Scatter chart. Label and format the chart accordingly.

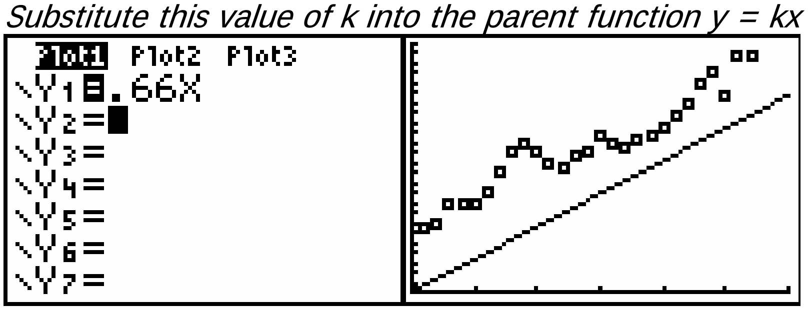




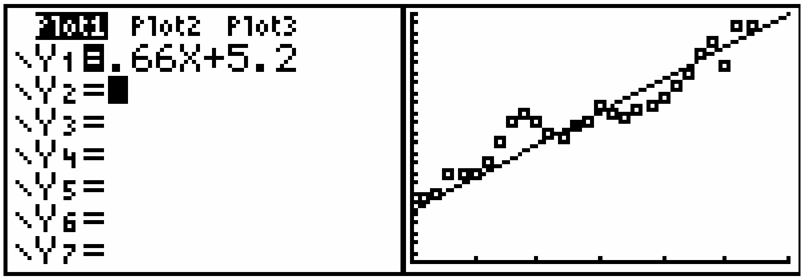
3. Ask students to share how they developed their function rule.
   Sample answer using a graphing calculator:
   The graph appears to have a linear trend. Find an average rate of change by looking at the first differences.



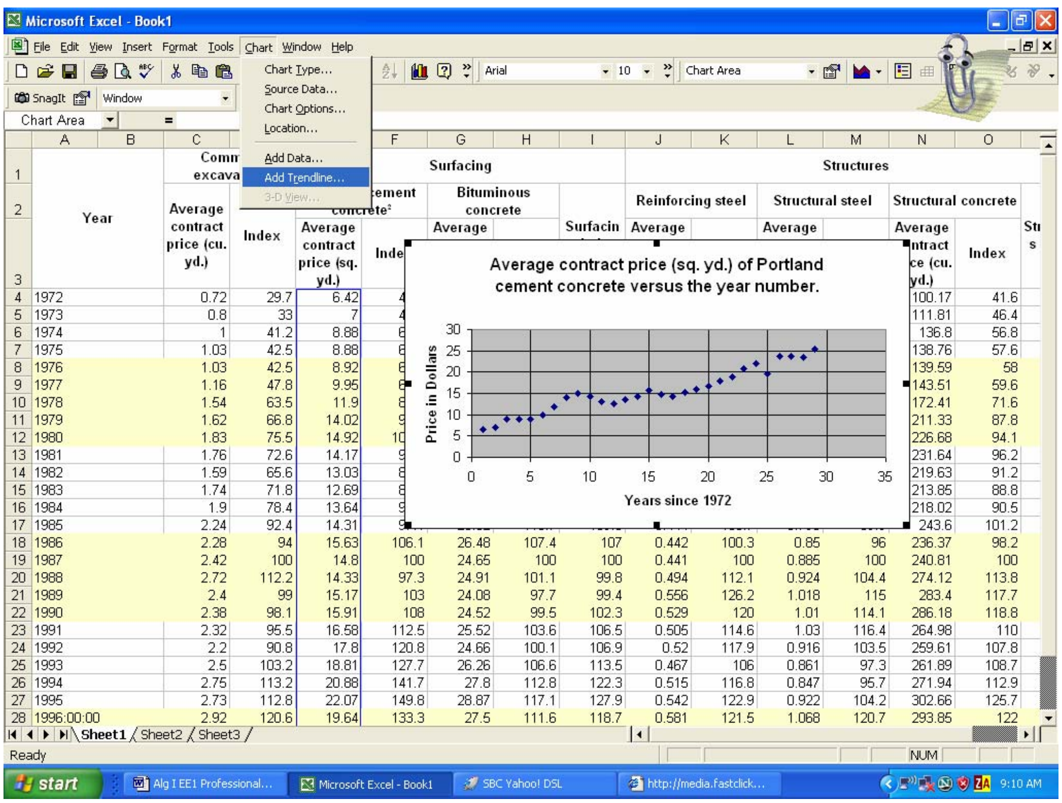
Substitute this value of k into the parent function y = kx and verify using a graph.



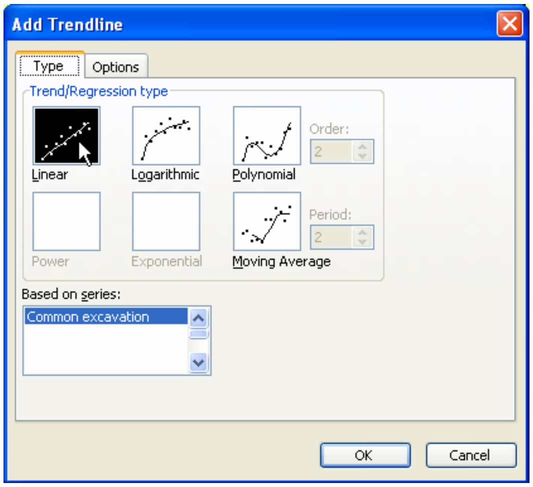
This function is not a good fit. Create a vertical shift of the graph by adding a y-intercept.



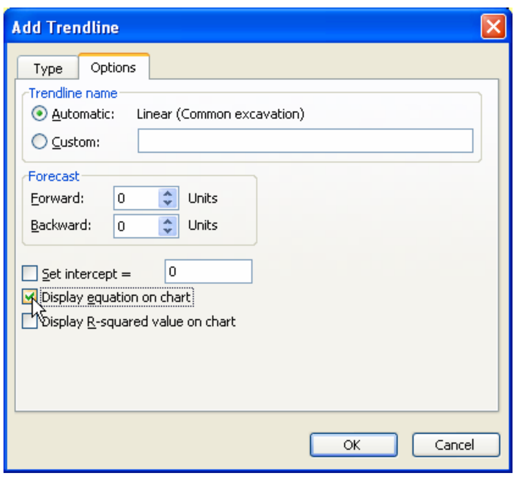
Sample answer using a spreadsheet in Excel:
From the Chart Menu, generate a scatterplot. From the Data menu, select Add Trendline.

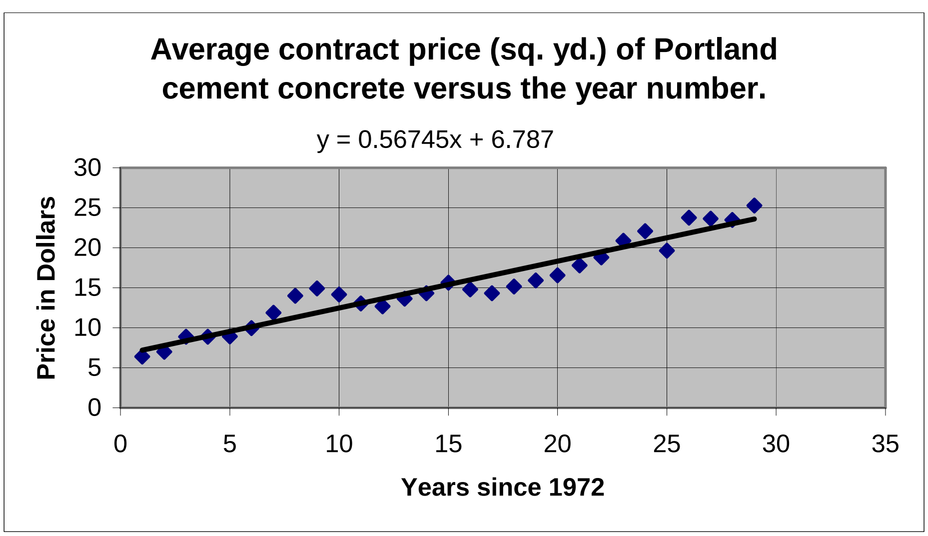


Since this is a linear function, choose Linear regression.



Be sure to select the "Display Equation on Chart" check box on the Options tab.



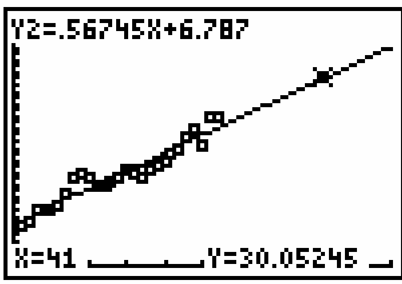


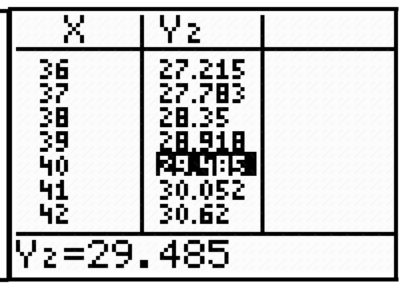
4. Ask students to share how they predicted the approximate cost of Portland cement concrete in the year 2012 and why they chose that method.
   Answers will vary depending on the data collected and method used to generate the function rule. Answers below are based on the linear regression model determined on the graphing calculator.

   2012 will be the 40th year since we considered 1972 to be year 0.

   Symbolically:
   $$y = .56745x + 6.787$$
   $$y = .56745(40) + 6.787$$
   $$y = 22.698 + 6.787$$
   $$y = 29.485$$

   Graphically or tabular:





   Using the rule, $y = .56745x + 6.787$ *, the average contract price (sq. yd.) of Portland cement concrete in the year 2012 will be approximately $29.49.*

ELABORATE

The Elaborate portion of the lesson provides an opportunity for the student to apply the concepts of the TEKS within a new situation. In this lesson, students will generate and compare different linear functions. This part of the lesson is designed for groups of three to four students.

1. Distribute the **Place Your Bids** activity sheet. Students should follow the directions to solve the problem.
2. Use the Facilitation Questions to redirect students as necessary**.**

Facilitation Questions – Elaborate Phase

- Is there a positive, negative, or no correlation between the two variables?
  Each of these relationships has a positive correlation.
- What is starting point?
  The starting point is the same as the cost of the item in 1972. This value can be found on the scatterplot or in the table in Excel or on the website.
- What is the approximate rate of change over time?
  This value can be found by taking the initial value and one of the final values (around year 27) and estimating the slope of the line connecting the two points.
- Is this curve a good fit? How can you adjust the slope or y-intercept to make it a better fit?
  Answers may vary. Students should be able to judge if their line needs to be more or less steep or if it should be translated up or down.
- What formula do you need to find the volume of this object?
  A cylinder's volume can be found using the formula $V = \pi r^2 h$. A half-cylinder would be half this value.
- What formula do you need to find the lateral surface area of the tunnel?
  The lateral surface area of a cylinder is SA = $2\pi rh$, so the lateral surface area of half-cylinder is πrh.

EVALUATE

The Evaluate portion of the lesson provides the student with an opportunity to demonstrate his or her understanding of the TEKS addressed in the lesson.

1. Provide each student with the **City Folk, Country Folk** activity sheet.
2. Upon completion of the activity sheet, a rubric should be used to assess student understanding of the concepts addressed in the lesson.

Answers and Error Analysis for selected response questions:

Question Number	*TEKS*	*Correct Answer*	*Conceptual Error*	*Conceptual Error*	*Procedural Error*	*Procedural Error*	*Guess*
1	A.2(D)	C	D	A			B
2	A.1(E)	C	A		B	D	
3	A.1(C)	B	A	D	C		
4	A.2(D)	D	A	B	C		

Road Construction

When you are planning a family road trip, many different factors need to be considered. A major consideration is the cost of gasoline for the family car. The cost of gasoline depends on the number of miles that will be driven, the fuel efficiency of the car, and the cost per gallon of gasoline. If the trip will be over several days, other factors such as food and lodging need to be taken into account.

City, regional, and state planners face similar decisions when planning highway construction projects. In this lesson, you will take the role of a city transportation planner. You will need to predict the cost of a major highway construction project in the year 2012.

1. **What kinds of costs are associated with a highway construction project?**
   Answers may vary. Students should include costs such as the cost of materials like concrete and steel, the cost of labor, and the cost of excavations and demolitions.

2. **What information would you need to make an accurate prediction of costs in 2012?**
   Answers may vary. Students should realize a need for price trends over past years.

3. **How could you use that information to make your prediction?**
   Answers may vary. Students should discuss analyzing the data using graphs, tables and function rules.

The Price of Concrete in Oregon

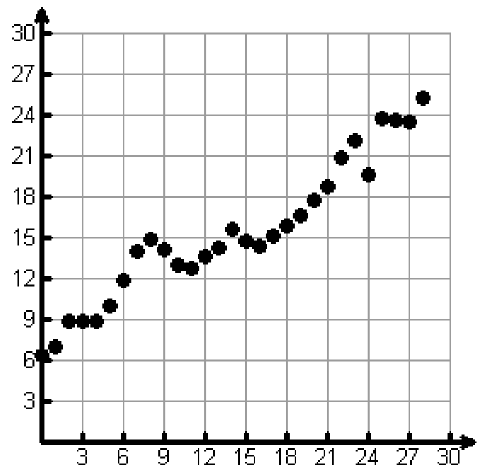
To make your prediction, you will explore price trends for federal-aid highway construction by visiting the U.S. Department of Transportation website for program administration, http://www.fhwa.dot.gov/programadmin/pt2004q1.htm. The website currently includes tables with yearly prices for different aspects of highway construction from 1972 to 2004. You will examine the different aspects such as excavation, surfacing and structural cost, to determine linear relationships. You will use the data to determine a function rule for an appropriate trend line. Then use your function rule to make predictions.

1. **Using the data from the website fill in the table below for the average contract price of Portland cement concrete. Describe the relationship.**
   Data for every other year was recorded and 1972 was entered as year 0.

Average Contract Price of Portland Cement

Years Since 1972	Price per square yard
0	*6.42*
2	*8.88*
4	*8.92*
6	*11.9*
8	*14.92*
10	*13.03*
12	*13.64*
14	*15.63*
16	*14.33*
18	*15.91*
20	*17.80*
22	*20.88*
24	*19.64*
26	*23.65*

2. **Generate a scatterplot of your data. Sketch your graph.**
   Actual scatterplots may vary depending on the technology chosen by the students.

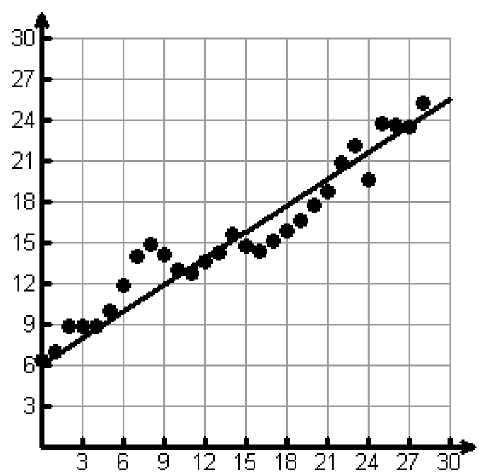


3. Does your scatterplot represent a positive, negative or no correlation? Explain your response.

The scatterplot represents a positive correlation. As the number of years increases the price increases.

4. Find an appropriate function rule for a trend line to model your data. Test the rule over your scatterplot. Write your function rule and sketch your graph.

Sample function rule: $y = 0.65x + 6$



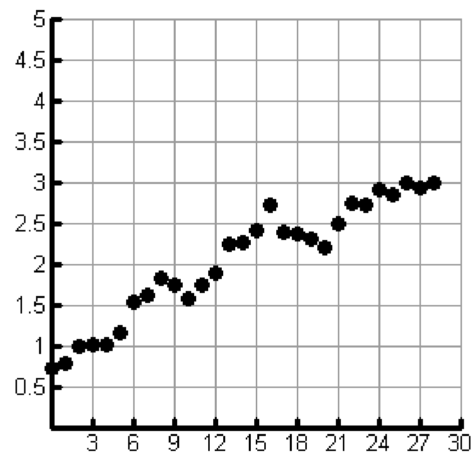
5. If these price trends continue predict the approximate cost of Portland cement concrete in the year 2012. Justify your answer.

Answers may vary. Using the rule, $y = 0.65x + 6$, the average contract price (sq. yd.) of Portland cement concrete in the year 2012 will be approximately \$32.00.

Place Your Bids

In 2012, the State Highway Department plans a new road construction project in the Davis Mountains in far West Texas. There is a mountain through which a tunnel must be dug and a road paved through.

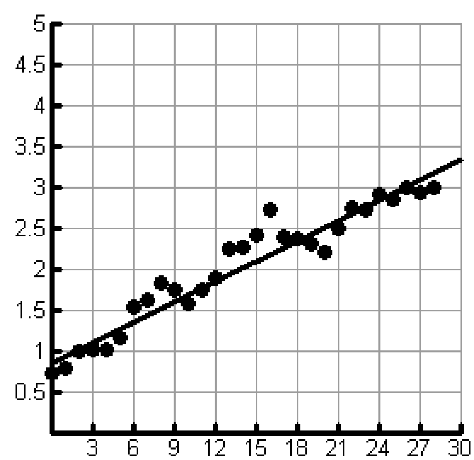
1. Using the Federal Highways Administration website from the previous activity, make a scatterplot of the average cost of common excavation vs. years since 1972. Sketch your scatterplot.



2. Generate a function rule to predict the average cost of common excavation in a given year. Graph your function rule over your scatterplot. Sketch your graph.

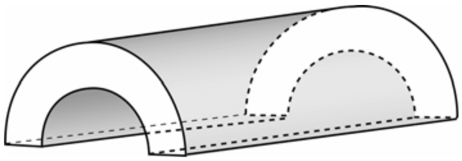
From the graph, a starting point of 0.75 and a change of about $2 in 24 years can be determined. Using transformations can refine the trend line.

Possible solution: $y = 0.083x + 0.85$



3. According to your function rule, how much will excavation cost in the year 2012?
2012 is 40 years after 1972, so y = 0.85 + 0.083(40) = $4.17 per cubic yard

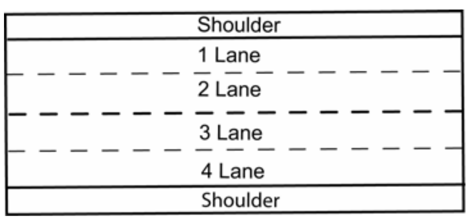
4. The tunnel will be in the shape of a half-cylinder (the base is a semi-circle) that is 1320 feet long with a radius of 25 feet.



a. **What is the approximate volume of the tunnel, in cubic yards?**
Using π = 3.14, the volume is about 1,295,250 cubic feet or 47,972.22 cubic yards

b. **How much will it cost to excavate the tunnel in 2012?**
About $200,044.17

5. The flat side of the tunnel will be the road surface. The road will consist of 4 lanes of traffic that are each 10 feet wide with a 5-foot wide shoulder on each side. The road will be paved with Portland cement concrete.



a. **What is your function rule from The Price of Concrete in Oregon activity used to determine the cost of Portland cement concrete?**
y = 0.65x + 6, where x represents the number of years since 1972.

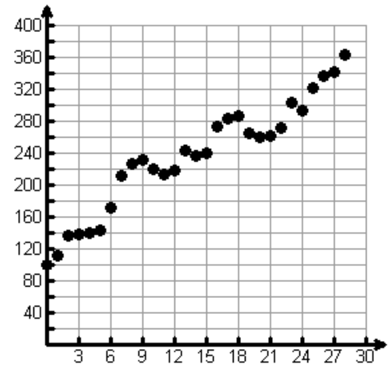
b. **Based on this function rule, what will the approximate cost of Portland cement concrete be in 2012?**
$32.00 per square yard

c. **In square yards, what is the approximate surface area of the road inside the tunnel?**
66,000 square feet or about 7,333 square yards

d. **How much will it cost to pave the road with Portland cement concrete in 2012?**
About $234,656

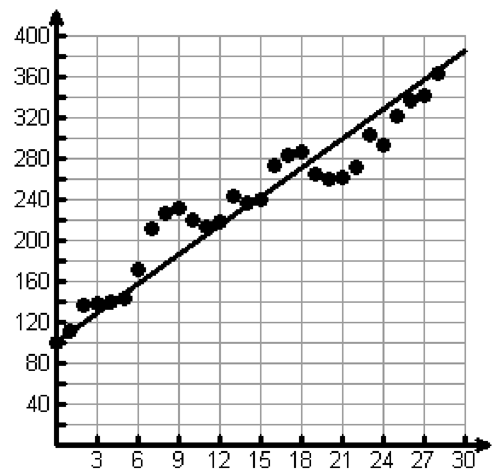
6. The sides and top of the tunnel will be paved with structural concrete.

a. **Using the same data table from the Federal Highways Administration, make a scatterplot of the average cost of structural concrete vs. years since 1972. Sketch your scatterplot.**



b. **Generate a function rule to predict the average cost of structural concrete in a given year. Graph your function rule over your scatterplot. Sketch your graph.**

Sample answer. y = 9.5x + 100



c. **According to your function rule, how much will structural concrete cost in the year 2012?**

2012 is 40 years after 1972, so y = 9.5(40) + 100 = $480 per cubic yard

7. The sides and top of the tunnel will be paved with structural concrete that is 6 inches thick.

a. **In square feet, what is the approximate lateral surface area of the sides and top of the tunnel? (**$L.A. = \frac{2\pi rh}{2}$**)**

Using π *= 3.14, the lateral surface area is about 103,620 square feet*

b. **In cubic feet, what is the volume of structural concrete that will be needed?**
Multiply the lateral surface area by .5 feet. Answer approximately 51,810 cubic feet

c. **In cubic yards, what is the volume of structural concrete that will be needed?**
About 1919 cubic yards

d. **What will be the cost of the structural concrete?**
About $921,120

8. What will be the total cost of the materials for the project?
Adding the cost of excavation, Portland cement concrete for the roadway and structural concrete for the walls of the tunnel, the total cost will be about $200,044.17 + $234,656 + $921,120 = $1,355,820.17

City Folk, Country Folk

Angelique Hereford is the current county commissioner for Alpha County. She recently learned that the state has given her county $3.25 million dollars towards concrete costs for new pavement on her county roads. Alpha County is mostly rural, but has a mid-size city near its center.

Commissioner Hereford would like to use the money for two projects. In the rural area, a 5-mile stretch of road needs to be paved. The road will be 30 feet wide, including shoulders. Inside the city, a 1-mile stretch of a 4-lane road desperately needs new pavement. The city road is 40 feet wide with curbs instead of shoulders. Engineers have advised her to use Portland cement concrete for both projects.

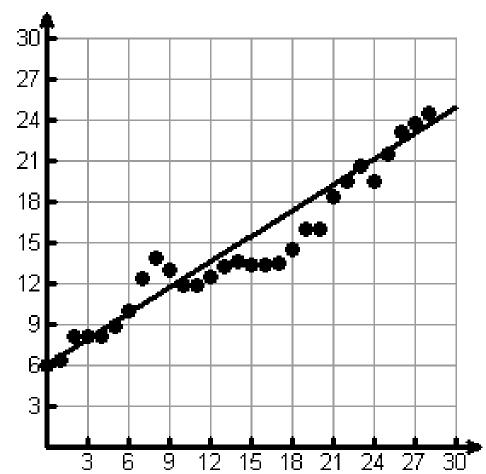
Commissioner Hereford consulted the Federal Highways Administration website to obtain construction materials' costs at the web address below:

http://www.fhwa.dot.gov/programadmin/pt2004q1.htm

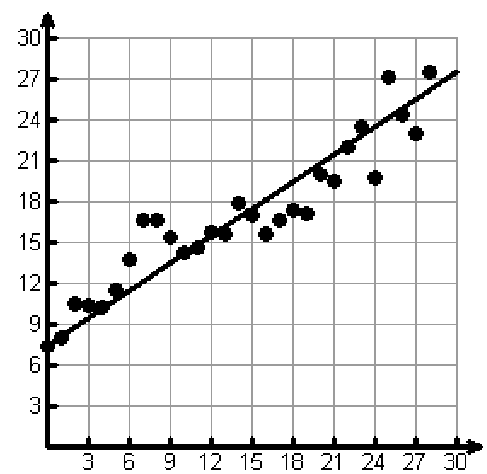
Based on the price trends on the second table, **Price Trends for Federal-aid Highway Construction Rural and Urban**, will Commissioner Hereford have enough money for both projects? Justify your answer.

Answer: Yes. Following are scatterplots and function rules for rural and urban costs of Portland cement concrete:

Rural Portland Cement, y = 0.63x +6



Urban Portland Cement, y = 0.67x +7.39



Based on these costs, the rural road will cost $2,357,520 in concrete and the urban road will cost $645,099 in concrete. The sum of these costs is less than $3.25 million.

Road Construction

When you are planning a family road trip, many different factors need to be considered. A major consideration is the cost of gasoline for the family car. The cost of gasoline depends on the number of miles that will be driven, the fuel efficiency of the car, and the cost per gallon of gasoline. If the trip will be over several days, other factors such as food and lodging need to be taken into account.

City, regional, and state planners face similar decisions when planning highway construction projects. In this lesson, you will take the role of a city transportation planner. You will need to predict the cost of a major highway construction project in the year 2012.

1. What kinds of costs are associated with a highway construction project?

2. What information would you need to make an accurate prediction of costs in 2012?

3. How could you use that information to make your prediction?

The Price of Concrete in Oregon

To make your prediction, you will explore price trends for federal-aid highway construction by visiting the U.S. Department of Transportation website for program administration, http://www.fhwa.dot.gov/programadmin/pt2004q1.htm. The website currently includes tables with yearly prices for different aspects of highway construction from 1972 to 2004. You will examine the different aspects such as excavation, surfacing and structural cost, to determine linear relationships. You will use the data to determine a function rule for an appropriate trend line. Then use your function rule to make predictions.

1. Using the data from the website, fill in the table below for the average contract price of Portland cement concrete. Describe the relationship.

Average Contract Price of Portland Cement

Years Since 1972	Price per square yard

2. Generate a scatterplot of your data. Sketch your graph.

3. Does your scatterplot represent a positive, negative or no correlation? Explain your response.

4. Find an appropriate function rule for a trend line to model your data. Test the rule over your scatterplot. Write your function rule and sketch your graph.

5. If these price trends continue predict the approximate cost of Portland cement concrete in the year 2012. Justify your answer.

Place Your Bids

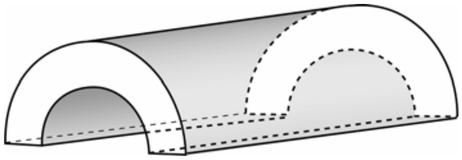
In 2012, the State Highway Department plans a new road construction project in the Davis Mountains in far West Texas. There is a mountain through which a tunnel must be dug and a road paved through.

1. Using the Federal Highways Administration website from the previous activity, make a scatterplot of the average cost of common excavation vs. years since 1972. Sketch your scatterplot.

2. Generate a function rule to predict the average cost of common excavation in a given year. Graph your function rule over your scatterplot. Sketch your graph.

3. According to your function rule, how much will excavation cost in the year 2012?

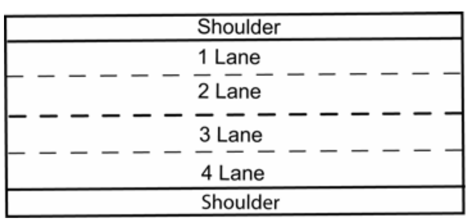
4. The tunnel will be in the shape of a half-cylinder (the base is a semi-circle) that is 1320 feet long with a radius of 25 feet.



   a. What is the approximate volume of the tunnel, in cubic yards?

   b. How much will it cost to excavate the tunnel in 2012?

5. The flat side of the tunnel will be the road surface. The road will consist of 4 lanes of traffic that are each 10 feet wide with a 5-foot wide shoulder on each side. The road will be paved with Portland cement concrete.



   a. What is your function rule from **The Price of Concrete in Oregon** activity used to determine the cost of Portland cement concrete?

   b. Based on this function rule, what will the approximate cost of Portland cement concrete be in 2012?

c. In square yards, what is the approximate surface area of the road inside the tunnel?

d. How much will it cost to pave the road with Portland cement concrete in 2012?

6. The sides and top of the tunnel will be paved with structural concrete.
   a. Using the same data table from the Federal Highways Administration, make a scatterplot of the average cost of structural concrete vs. years since 1972. Sketch your scatterplot.

   b. Generate a function rule to predict the average cost of structural concrete in a given year. Graph your function rule over your scatterplot. Sketch your graph.

   c. According to your function rule, how much will structural concrete cost in the year 2012?

7. The sides and top of the tunnel will be paved with structural concrete that is 6 inches thick.
   a. In square feet, what is the approximate lateral surface area of the sides and top of the tunnel ($L.A. = \frac{2\pi rh}{2}$)?

   b. In cubic feet, what is the volume of structural concrete that will be needed?

   c. In cubic yards, what is the volume of structural concrete that will be needed?

   d. What will be the cost of the structural concrete?

8. What will be the total cost of the materials for the project?

City Folk, Country Folk

Angelique Hereford is the current county commissioner for Alpha County. She recently learned that the state has given her county $3.25 million dollars towards concrete costs for new pavement on her county roads. Alpha County is mostly rural, but has a mid-size city near its center.

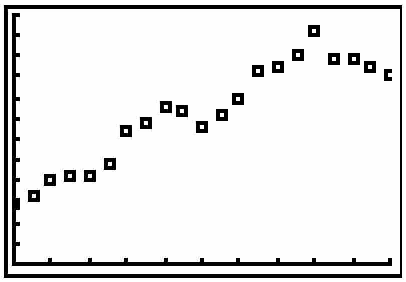
Commissioner Hereford would like to use the money for two projects. In the rural area, a 5-mile stretch of road needs to be paved. The road will be 30 feet wide, including shoulders. Inside the city, a 1-mile stretch of a 4-lane road desperately needs new pavement. The city road is 40 feet wide with curbs instead of shoulders. Engineers have advised her to use Portland cement concrete for both projects.

Commissioner Hereford consulted the Federal Highways Administration website to obtain construction materials' costs at the web address below:

http://www.fhwa.dot.gov/programadmin/pt2004q1.htm

Based on the price trends on the second table, **Price Trends for Federal-aid Highway Construction Rural and Urban**, will Commissioner Hereford have enough money for both projects? Justify your answer.

1 The scatterplot below shows the Index value for common excavation costs of road construction versus the number of years since 1972.



Which of the following describes this relationship?

A No correlation

B Neutral correlation

C Positive correlation

D Negative correlation

2 The table below shows the average cost of a roundtrip airline ticket from Houston to Dallas versus the number of years since 1999.

Number of years since 1999	Cost in Dollars
0	96.25
1	119.52
2	142.79
3	166.06
4	189.33
5	212.60
6	235.87

If this trend continues, what will be the average cost of a roundtrip airline ticket from Houston to Dallas in 2007?

A $240.10

B $259.14

C $282.41

D $305.68

3 The table below shows the total construction cost of a 100 square foot wood deck versus the number of years since 1999.

Number of years since 1999	Cost in Dollars
0	827.35
1	852.55
2	877.75
3	902.95
4	928.15
5	953.35
6	978.55

Which function rule best describes this relationship?

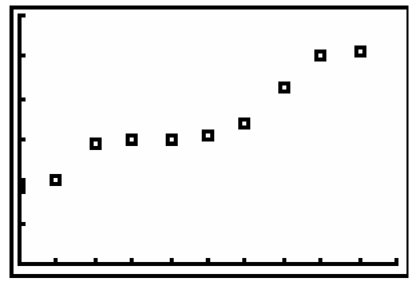
A $25.2x$

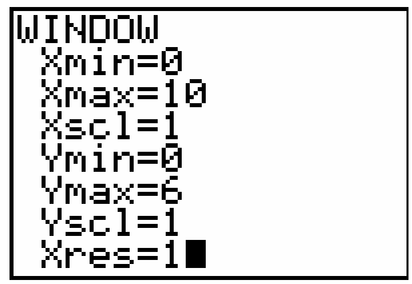
B $25.2x + 827.35$

C $2.2\,x + 827.35$

D $827.352x$

4 The scatterplot below was graphed in the window shown and shows the cost of one gallon of milk versus the number of years since 1995.





If this trend continues, what will be the average cost of a gallon of milk in 2010?

A $1.77

B $3.10

C $5.33

D $7.11

Motion and Quadratic Transformations

A1(B)	gather and record data and use data sets to determine functional relationships between quantities.
A1(C)	describe functional relationships for given problem situations and write equations or inequalities to answer questions arising from the situations;
A1(D)	represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and
A1(E)	interpret and make decisions, predictions, and critical judgments from functional relationships.
A2(B)	identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete.
A2(D)	collect and organize data, make and interpret scatterplots (including recognizing positive, negative, or no correlation for data approximating linear situations), and model, predict, and make decisions and critical judgments in problem situations.
A9(A)	determine the domain and range for quadratic functions in given situations.
A9 (B)	investigate, describe, and predict the effects of changes in a on the graph of $y = ax^2 + c$.
A9 (C)	investigate, describe, and predict the effects of changes in c on the graph of $y = ax^2 + c$.
A9(D)	analyze graphs of quadratic functions and draw conclusions.
A10(A)	solve quadratic equations using concrete models, tables, graphs, and algebraic methods.
A10(B)	make connections among the solutions (roots) of quadratic equations, the zeros of their related functions, and the horizontal intercepts (x-intercepts) of the graph of the function.

Materials

Advanced Preparation:

- Copies of activity sheets
- Assemble materials shown below for student groups

For each student:

- Graphing calculator
- **The Rainforest Canopy Tour** activity sheet
- **Canopy Tours and Slopes of Zip Lines** activity sheet
- **Canopy Tours and Length of Zip Lines** activity sheet
- **Down Hill Racing** activity sheet

For each student group of 3 - 4 students:

- 2 meter sticks,
- About 4 meters of a vinyl clothesline (plastic coated works better than nylon-ply),
- Pulley,
- Action figure,
- 2 large rubber bands (#64 postal size works well),
- Tape measure (metric),
- CBR,
- Graphing calculator,
- Linking cable

ENGAGE

The Engage portion of the lesson is designed to create student interest in the relationship between distance and the time it takes to zip from platform to platform on a Canopy Tour. Students should work in groups of three to four.

1. *Begin the activity by showing the short canopy tour video found at* www.monteverdetours.com/tours/monteverde/sky_trek_video_org.htm
2. Distribute **The Rainforest Canopy Tour** activity sheet. Students should read the passage and answer the questions on the activity sheet.
3. When most students have completed the activity sheet, prompt them to discuss their predictions and what knowledge they have learned with group members.
4. Debrief the activity using the Facilitation Questions.

Facilitation Questions – Engage Phase

1. **If a person is traveling along a sloped zip line do you think they will speed up, slow down, or travel at a constant rate? Why do you think so?**
   They will speed up as long as the slope is steep enough to cause motion by overcoming friction.
2. **What happens to the person's speed if the slope of the zip line gets steeper?**
   They go faster.
3. **What are the characteristics of a distance versus time graph created by an object that is traveling at a constant rate?**
   It will be a linear graph.
4. **What are the characteristics of a distance versus time graph created by an object that is not traveling at a constant rate?**
   The graph will curve.

EXPLORE

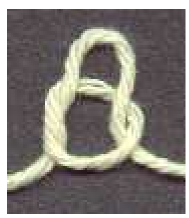
The Explore portion of the lesson provides the student with an opportunity to be actively involved in the exploration of the mathematical concepts addressed. In this activity students will simulate a rain forest canopy tour experience using meter sticks, a vinyl line and pulley, and an action figure. They will use a CBR to collect data and generate a quadratic function rule describing the motion of an action figure as it slides down the vinyl line. By varying the slope of the vinyl "zip line," students will investigate changes in the parameter a in the function $y = ax^2 + c$. Students will connect the real-world physics of motion to the function rule and use the function rule to solve real-world problems related to the design of a rain forest canopy tour experience. This part of the lesson is designed for groups of three to four students.

1. Distribute the **Canopy Tours and Slopes of Zip Lines** activity sheet.

   Demonstrate the use of the rubber bands to attach the action figure to the pulley. Next, thread the zip line through the pulley and tie each end of the zip line to the meter sticks using slipknots. The wheel on the pulley must spin freely. A lubricant like WD 40 will help it spin. Also you may need to demonstrate how to make a slipknot.

1. Start by looping the chord as shown.

2. Pull the cord through the loop.



3. Insert the meter stick.

   Ask 4 students to help you demonstrate the set up of the experiment. Explain the role of each student as listed on the activity page. Emphasize that the zip line must be pulled tight while keeping the meter sticks perpendicular to the floor. Also the release of the action figure and the start of data collection must be consistent for each trial. Position the CBR slightly to the side of the meter stick so the action figure will not hit the CBR.

2. Have students follow the directions on the activity sheet to collect data and explore the relationship between slopes of zip lines and distance versus time function rules.

Facilitation Questions – Explore Phase

- **Why do you think the function is nonlinear?**
  The person will be accelerating as they travel on the zip line.
- **How will the velocity of the action figure change when you change the slope of the zip line?**
  The steeper the zip line, the faster the person will travel.
- **What information do you need to calculate slope?**
  The distance between the meter sticks and the change in height from beginning point to end point.

EXPLAIN

The Explain portion of the lesson is directed by the teacher to allow the students to formalize their understanding of the TEKS addressed in the lesson.

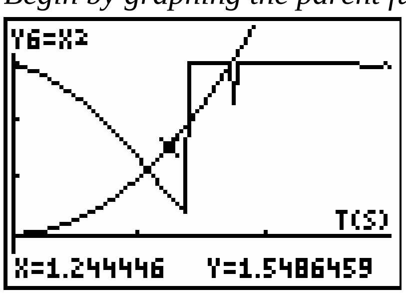
1. Debrief the **Canopy Tour** activity sheet. Use the Facilitation Questions to help students make connections to understand the role of *a* and *c* in $y = ax^2 + c$. Have each student group present the way they solved the problems.

Facilitation Questions – Explain Phase

1. How did you determine the slope of your zip line?
Answers may vary. Students may have measured the distance along the floor to get the "run" and found the difference between the low and high settings on the meter sticks to get the "rise."

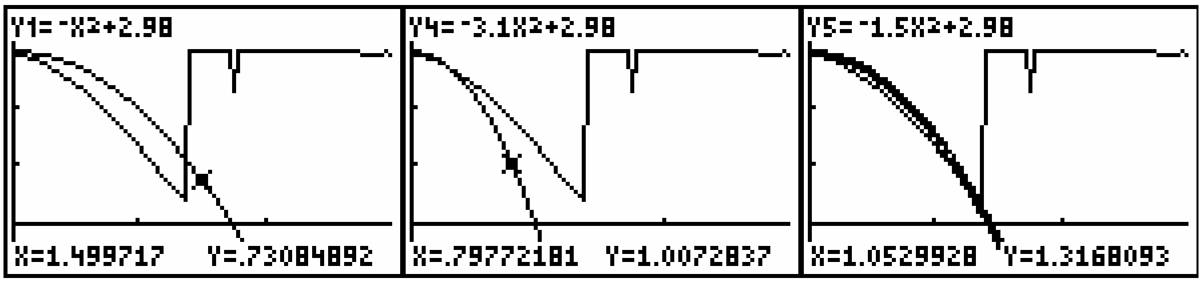
How did you determine a function rule to fit your data? Why did you choose this method?
Ask students to share their methods and their reasons for making that choice. If none of the participants choose a graphing calculator, be sure to explain its use.

Sample answer using a graphing calculator:
Begin by graphing the parent function $y = x^2$.



Facilitation Questions – Explain Phase-continued

To create a reflection of the function $y = x^2$ *change the coefficient of* x^2 *to its opposite. Also, since the distance at time 0 was 2.98 meters, add 2.98 to the function. This gives you the function* $y = -x^2 + 2.98$ *. Next try different values for the coefficient of* x^2 *until you have a good fit.*

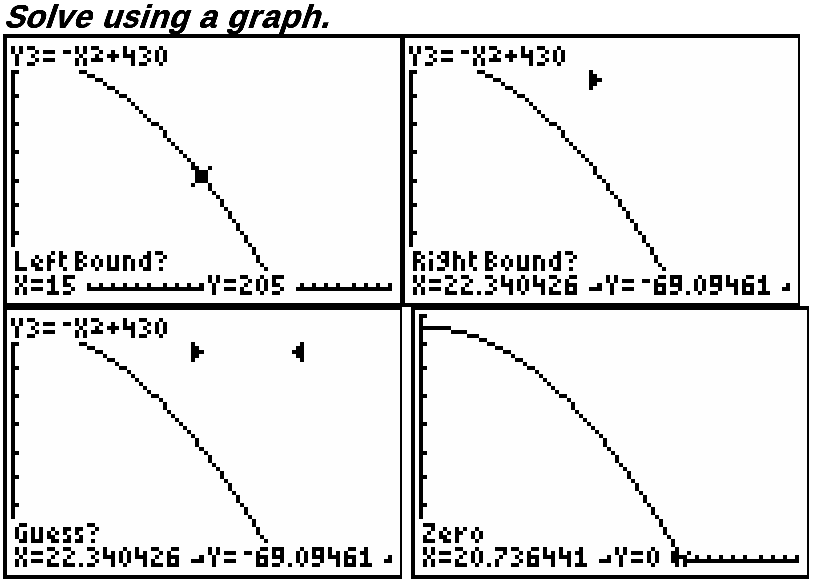


3. How did you solve the problem?

The Rainforest Canopy Tour Company advertises it longest zip line to be 430 meters from platform to platform. If the slope of this zip line is the same as your slope in phase 2 of this experiment, about how much time should it take Jose to zip along the line from one platform to the other?

Answers will vary. Answers below are based on the Phase 2 function rule $y = -x^2 + 2.98$ *. Since the slope of the zip line is the same as the slope of the zip line in phase 2, the coefficient of* x^2 *will be –1. In phase 2 the distance at time 0 was 2.98 meters. The distance at time 0 in the problem is 430 meters. Therefore, the function* $y = -x^2 + 430$ *describes this situation.*

Solve using a graph.



Facilitation Questions – Explain Phase-continued

Solve using a table.

X	Y3
20.7	1.51
20.71	1.0959
20.72	.6816
20.73	.2671
20.74	-.1476
20.75	-.5625
20.76	-.9776

X=20.74

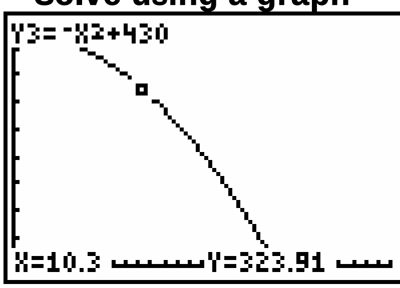
Solve symbolically

$$y = -x^2 + 430$$
$$0 = -x^2 + 430$$
$$x^2 = 430$$
$$x = \sqrt{430}$$
$$x \approx 20.74$$

4. How did you solve the problem?

When Jose travels along the 430-meter line, about how far will he be from the next platform after 10.3 seconds?

Solve using a graph



Solve using a table

X	Y3
10	330
10.1	327.99
10.2	325.96
10.3	323.91
10.4	321.84
10.5	319.75
10.6	317.64

Y3=323.91

ELABORATE

The Elaborate portion of the lesson provides an opportunity for the student to apply the concepts of the TEKS within a new situation. In this activity students will simulate a Canopy tour. They will use a CBR to collect data and generate a quadratic function rule describing the motion of an action figure as it slides down the vinyl line. By keeping the slope of the vinyl "zip line" constant while changing the starting point of the action figure, students will investigate changes in the parameter c in the function $y = ax^2 + c$. Students will connect the real-world physics of motion to the function rule and use the function rule to solve real-world problems related to the design of a rain forest canopy tour experience. This part of the lesson is designed for groups of three to four students.

1. Distribute the **Canopy Tours and Length of Zip Lines** activity sheet.
2. Have students follow the directions on the activity sheet to collect data and explore the relationship between starting points on the zip line and distance versus time function rules..
3. Debrief the **Canopy Tours and Length of Zip Lines** activity sheet.

Facilitation Questions – Elaborate Phase

1. **How will the distance traveled affect the speed of the action figure?**
   Students should realize that the action figure would travel faster as distance increases.
2. **Why is the slope of the zip line the same regardless of starting point?**
   As the action figure moves along the zip line the ratio $\frac{rise}{run}$ *remains the same. Properties of similar triangles can be explored at this time.*
3. **How do changes in the starting point affect the function rule?**
   It causes a change in c in the function rule $y = ax^2 + c$
4. **How do changes in the starting point affect the graph of the function?**
   It causes a vertical translation.

EVALUATE

The Evaluate portion of the lesson provides the student with an opportunity to demonstrate his or her understanding of the TEKS addressed in the lesson.

1. Provide each student with the **Down Hill Racing** activity sheet.
2. When students complete the activity sheet, the teacher should use a rubric to assess student understanding of the concepts addressed in the lesson.

Answers and Error Analysis for selected response questions:

Question Number	*TEKS*	*Correct Answer*	*Conceptual Error*	*Conceptual Error*	*Procedural Error*	*Procedural Error*	*Guess*
1	A.9(C)	B	A	C	D		
2	A.1(E)	C	B	A			D
3	A.9(D)	A	C	B	D		
4	A.1(B)	B	A	C			D

The Rainforest Canopy Tour

Tropical rain forests are dense, lush forests located in the tropical regions of the Earth- Central America, northern South America, central Africa, and southeast Asia. Large, tall trees, growing well over 100 feet tall, form the top layer of the rain forest. Their leaves and branches form what is called the "canopy" of the rain forest, shading the forest floor below.

Some entrepreneurs developed what they call "Canopy Tours." If you go on a Canopy Tour, you will be tied in with a harness then sent at high speeds along wire "zip lines" from treetop to treetop, landing on platforms constructed in the treetops.

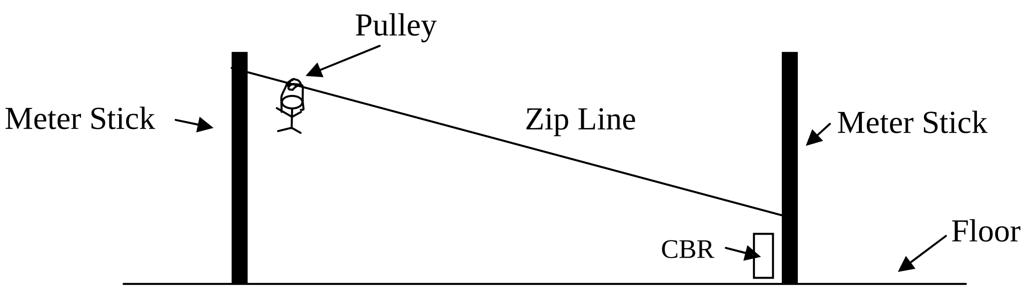
Just how long does it take to travel from one treetop to another? One way to determine the time is to examine the relationship between time and distance traveled.

1. **Imagine stepping off a platform 30 meters above the forest floor and zipping 300 meters along a zip line to the next platform 25 meters above the forest floor. Predict and sketch a distance versus time graph of your journey.**
   Answers may vary. Some students may sketch linear functions and some may sketch quadratic functions.

2. **Suppose, instead of 25 meters, the second platform were 20 meters above the forest floor. How would this change your graph from the graph in question 1? Sketch your new graph.**
   Answers may vary. Students should realize that this graph should have a greater rate of change than the first.

When you design a canopy tour, you must predict the time it will take to travel from platform to platform. You can make predictions using function rules.

To determine these function rules, you will set up an experiment to simulate a person traveling along a zip line from platform to platform high above the floor of the rainforest. You will collect data using a CBR then use your data to determine a function rule.



Set up the experiment as shown so that your zip line is about 3 meters long.

Canopy Tours and Slopes of Zip Lines

To conduct this experiment, your group members will need to assume the following roles:

- **2 Meter Stick Managers** pull the zip line tight while keeping the meter stick perpendicular to the floor.
- **Release Manager** holds the pulley on the zip line, against the meter stick. When the CBR operator says “Go,” he/she releases the pulley.
- **CBR Operator** starts data collection when he/she says, “Go.”

Data Collection Phase 1

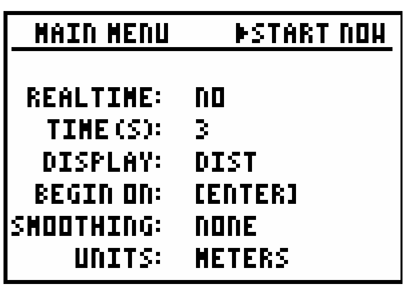
1. **Place one end of the zip line 20 centimeters above the floor and the other end 100 centimeters above the floor. Pull the zip line tight while keeping the meter sticks perpendicular to the floor. What is the distance between your meter sticks?**

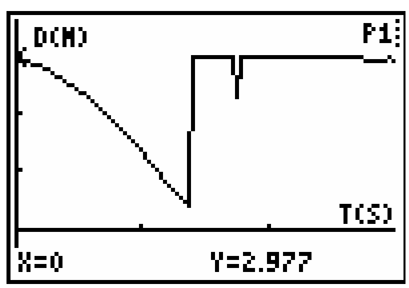
   For our sample data: 296 centimeters.

2. **What is the slope of your zip line?**

   For our sample data: $\frac{80\ cm}{296\ cm} \approx 0.270$

3. **Collect the data. Sketch the graph from your calculator.**

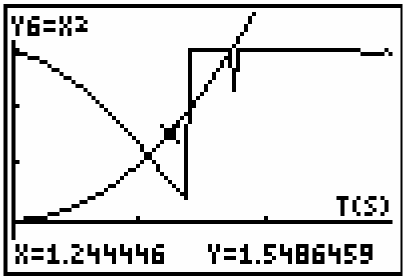




4. **What type of relationship does your graph appear to represent? How do you know?**
   Quadratic, since the curve becomes steeper as time increases.

5. **What is the parent function for this type of relationship?**
   $y = x^2$

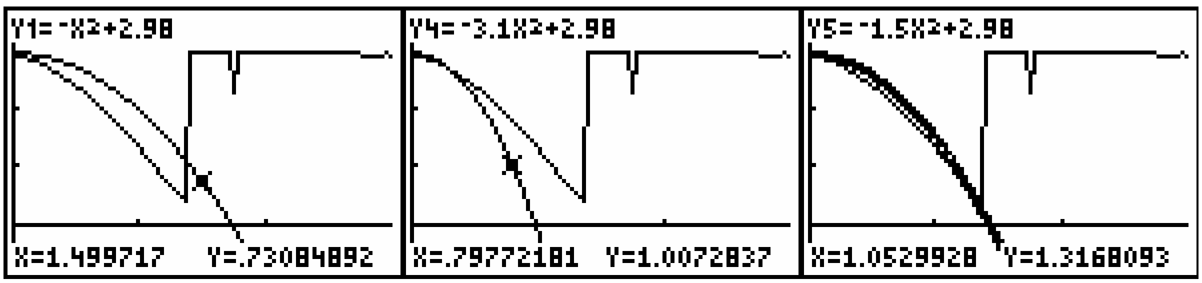
6. **Graph the parent function over your scatterplot. Sketch your results.**



7. **Is the parent function a good fit to your data? Why or why not?**

   The function rule is not a good fit. The graph should curve downward and shift vertically.

8. **Adjust your function rule until you have a curve that fits the data. Write your rule and sketch your graph.**



9. **In phase 2 of this experiment, you will decrease the slope of the zip line. Predict how the decrease in slope will affect your function rule and your graph.**
   Write your predicted function rule and sketch your predicted graph.

   Students' sketches should show a "less steep" curve, and the absolute value of the coefficient of x^2 should be less than the absolute value of the coefficient of x^2 in the function rule from question 8.

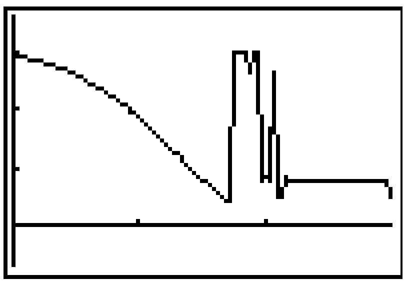
Data Collection Phase 2

1. **Place one end of the zip line 20 centimeters above the floor and the other end 85 centimeters above the floor. Pull the zip line tight while keeping the meter sticks perpendicular to the floor. What is the distance between your meter sticks?**

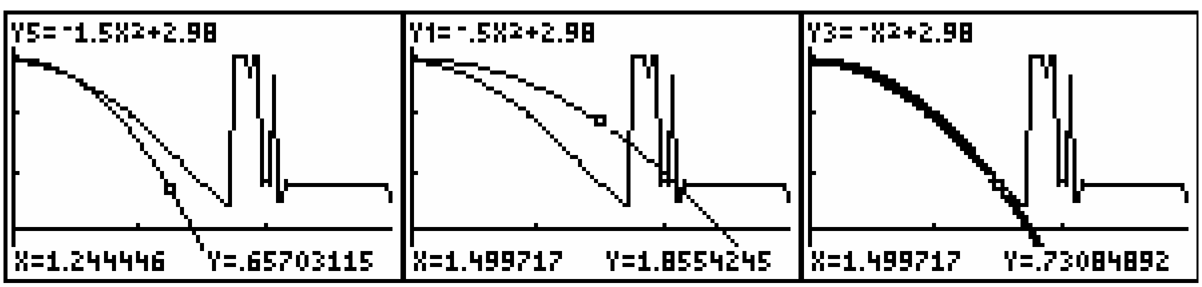
   For our sample data: 300 centimeters

2. **What is the slope of your zip line?**
   $\frac{65cm}{300\,cm} \approx 0.217$

3. **Collect the data. Sketch the graph from your calculator.**



4. **Determine a function rule that fits your data. Write your rule and sketch your graph.**



5. **In phase 3 of this experiment, you will further decrease the slope of the zip line. Predict how this decrease in slope will affect your function rule and your graph. Write your predicted function rule and sketch your predicted graph.**

   Students' sketches should show a "less steep" curve, and the absolute value of the coefficient of x^2 *should be less than the one in the function rule in question 13.*

Data Collection Phase 3

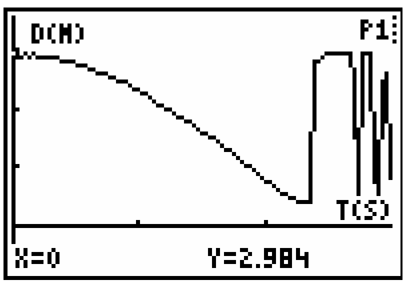
1. **Place one end of the zip line 20 centimeters above the floor and the other end 70 centimeters above the floor. Pull the zip line tight while keeping the meter sticks perpendicular to the floor. What is the distance between your meter sticks?**

   For our sample data: 303 centimeters

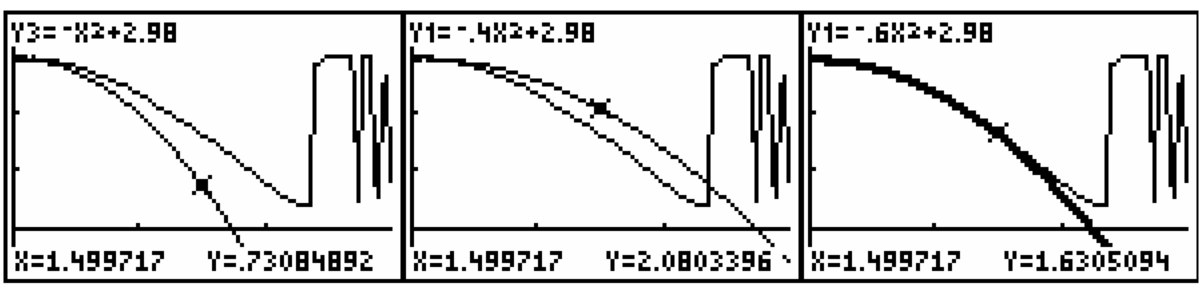
2. **What is the slope of your zip line?**

   $$\frac{50cm}{303\,cm} \approx 0.165$$

3. **Collect the data. Sketch the graph from your calculator.**

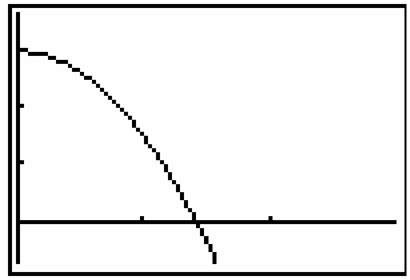
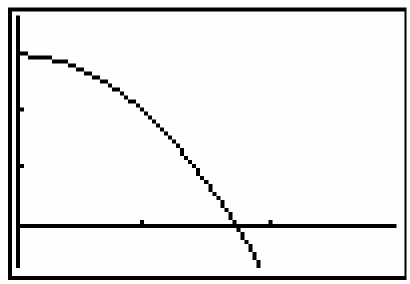
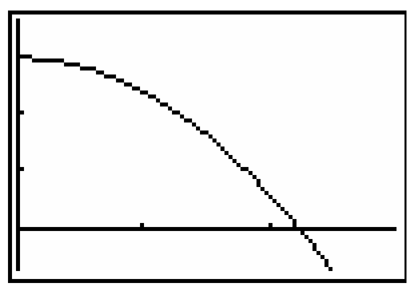


4. **Determine a function rule that fits your data. Write your rule and sketch your graph.**



Making Connections

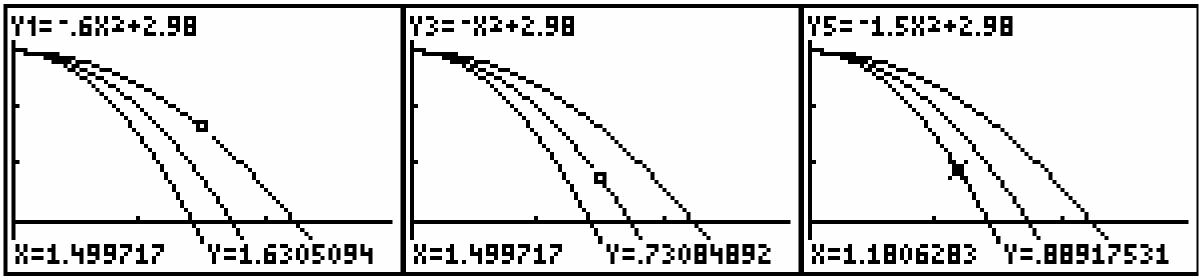
1. Summarize your findings in the table below.

	Slope of the Zip Line	Function Rule	Graph
Phase #1	*0.270*	$y = -1.5x^2 + 2.98$	
Phase #2	*0.217*	$y = -x^2 + 2.98$	
Phase #3	*0.165*	$y = -0.6x^2 + 2.98$	

2. How are the function rules alike? What accounts for these similarities?
Each rule contains a starting point of 2.98 because the action figure started the same distance away from the motion detector for each phase.

3. How are the function rules different? What accounts for these differences?
The coefficients of x^2 *are different because the action figure accelerates at different rates.*

4. Graph all three function rules in the same window. Sketch your graph.



5. How are the graphs alike? What accounts for these similarities?

Sample Observations: They all curve because as time increases the action figure speeds up. They all have the same y-intercept because the action figure started the same distance away from the motion detector for each phase.

6. How are the graphs different? What accounts for these differences?

They curve differently because the action figure accelerates at different rates.

7. In the context of this experiment, what is the meaning of the y-intercept in each of your graphs?

The y-intercept is the distance in meters the action figure was from the motion detector when it started down the zip line.

8. In the context of this experiment, what is the meaning of the x-intercept in each of your graphs?

The x-intercept is the time in seconds that it took the action figure to travel from one end of the zip line to the other.

9. In general what is the effect on the value of a in the function rule $y = ax^2 + c$ when the slope of the zip line decreases? Why does this happen?

The absolute value of a decreases because the action figure accelerates more slowly.

10. What happens to the function rule as the slope of the zip line approaches 0?

As the slope approaches 0, the value of a approaches 0 as well. So the function rule becomes $y = 0x^2 + c$ *or the constant function* $y = c$ *.*

11. In general what is the effect on the value of *a* in the function rule $y = ax^2 + c$ when the slope of the zip line increases? Why does this happen?

The absolute value of a increases because the action figure accelerates faster.

12. What happens to the function rule as the zip line becomes vertical?

As the zip line becomes more vertical the path of the action figure approaches free fall motion. Since the acceleration due to gravity is –9.8 meters per second squared the function rule becomes $y = -4.9x^2 + c$.

13. The Rainforest Canopy Tour Company advertises it longest zip line to be 430 meters from platform to platform. If the slope of this zip line is the same as your slope in phase 2 of this experiment, about how much time should it take Jose to zip along the line from one platform to the other?

It should take about 20.74 seconds to travel 430 meters.

14. When Jose travels along the 430-meter line, about how far will he be from the next platform after 10.3 seconds?

323.91 meters

Canopy Tours and Length of Zip Lines

To conduct this experiment your group members will need to assume the following roles:

- **2 Meter Stick Managers** pull the zip line tight while keeping the meter stick perpendicular to the floor.
- **Release Manager** holds the pulley on the zip line, at different distances from the meter stick. When the CBR operator says "Go," he/she releases the pulley.
- **CBR Operator** starts data collection when he/she says, "Go."

Data Collection Phase 1

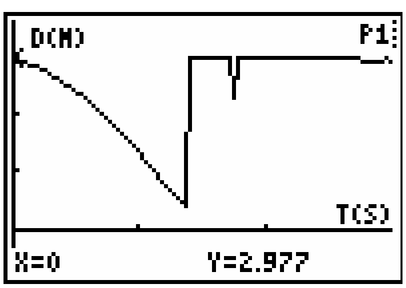
1. **Place one end of the zip line 20 centimeters above the floor and the other end 100 centimeters above the floor. Pull the zip line tight while keeping the meter sticks perpendicular to the floor. What is the distance between your meter sticks?**

   For our sample data: 296 centimeters.

2. **What is the slope of your zip line?**

   For our sample data: $\frac{80\ cm}{296\ cm} \approx 0.270$

3. **Collect the data. For this phase of the data collection, the Release Manager needs to start by holding the pulley against the meter stick and releasing it from that position. Sketch the graph from your calculator.**

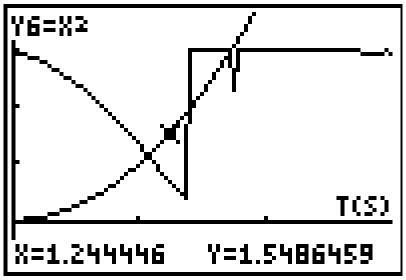


4. **What type of relationship does your graph appear to represent? How do you know?**
   Quadratic, since the curve becomes steeper as time increases.

5. What is the parent function for this type of relationship?

$y = x^2$

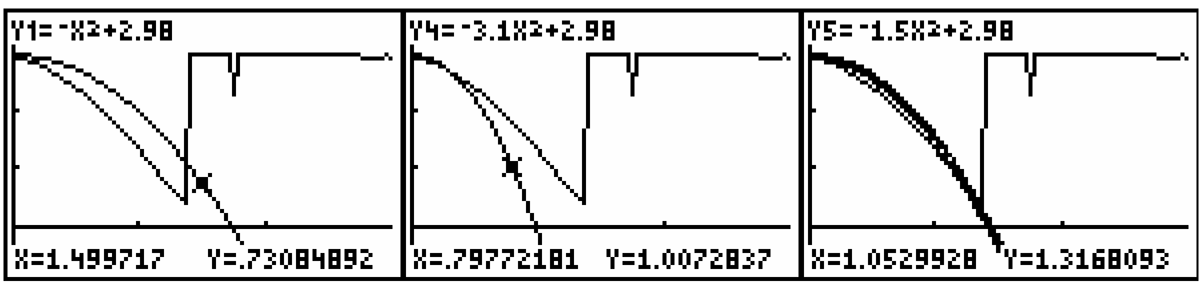
6. Graph the parent function over your scatterplot. Sketch your results.



7. Is the parent function a good fit to your data? Why or why not?

The function rule is not a good fit. The graph should curve downward and shift vertically.

8. Adjust your function rule until you have a curve that fits the data. Write your rule and sketch your graph.



9. In phase 2 of this experiment, you will decrease the distance your action figure travels on the zip line. Predict how the decrease in distance will affect your function rule and your graph.
Write your predicted function rule and sketch your predicted graph.

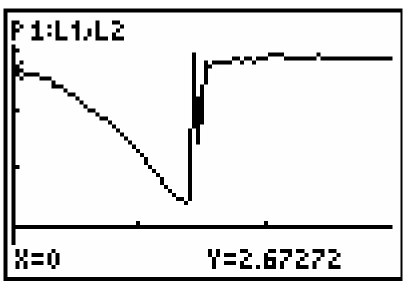
Students' sketches should show a similar curve. The value of the coefficient of x^2 *should be the same the coefficient of* x^2 *in the function rule from question 8; however, the value of c in the rule* $y = x^2 + c$ *should decrease.*

Data Collection Phase 2

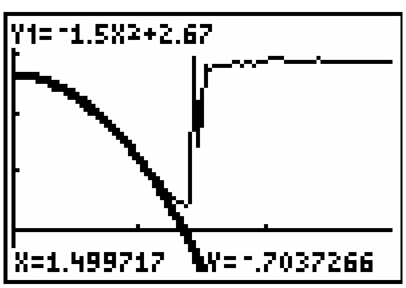
1. **Once again, place one end of the zip line 20 centimeters above the floor and the other end 100 centimeters above the floor. Pull the zip line tight while keeping the meter sticks perpendicular to the floor. What is the distance between your meter sticks?**
   For our sample data: 296 centimeters.

2. **What is the slope of your zip line?**
   For our sample data: $\frac{80\,cm}{296\,cm} \approx 0.270$

3. **Collect the data. For this phase of the data collection, the Release Manager needs to start by holding the pulley one foot from the meter stick and releasing it from that position. Sketch the graph from your calculator.**



4. **Determine a function rule that fits your data. Write your rule and sketch your graph.**



5. **In phase 3 of this experiment, you will further decrease the distance your action figure travels on the zip line. Predict how the decrease in distance will affect your function rule and your graph.**
   Write your predicted function rule and sketch your predicted graph.

   Students' sketches should show a similar curve. The value of the coefficient of x^2 *should be the same the coefficient of* x^2 *in the function rule from question 13; however, the value of c in the rule* $y = x^2 + c$ *should decrease.*

Data Collection Phase 3

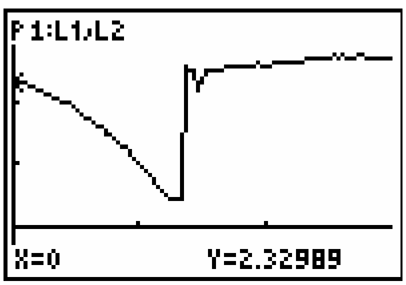
1. **Once again, place one end of the zip line 20 centimeters above the floor and the other end 100 centimeters above the floor. Pull the zip line tight while keeping the meter sticks perpendicular to the floor. What is the distance between your meter sticks?**

   For our sample data: 296 centimeters.

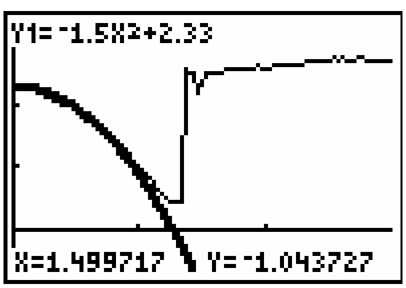
2. **What is the slope of your zip line?**

   For our sample data: $\frac{80\text{ cm}}{296\text{ cm}} \approx 0.270$

3. **Collect the data. For this phase of the data collection, the Release Manager needs to start by holding the pulley two feet from the meter stick and releasing it from that position. Sketch the graph from your calculator.**

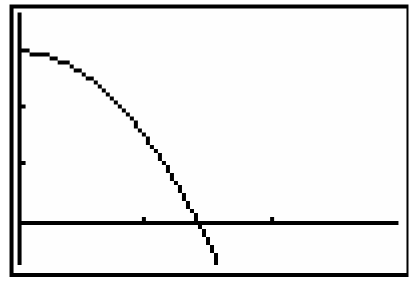
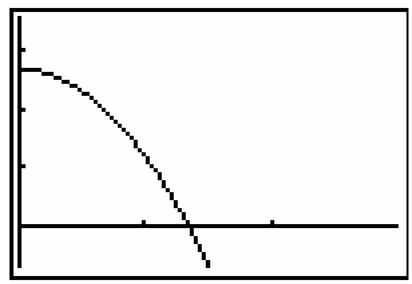
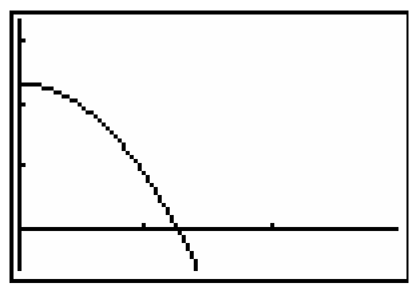


4. **Determine a function rule that fits your data. Write your rule and sketch your graph.**



Making Connections

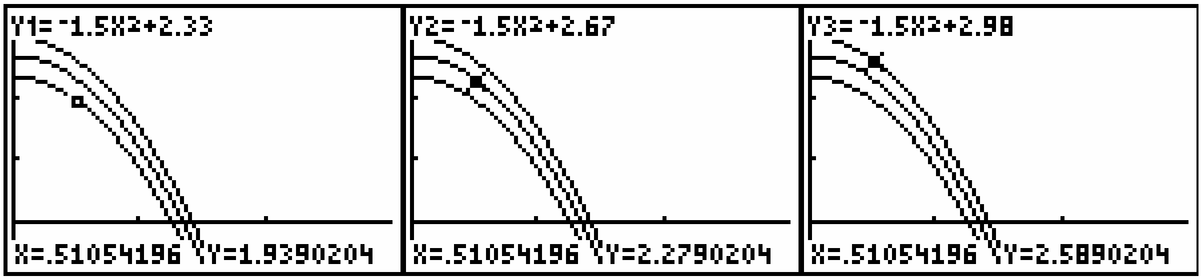
1. Summarize your findings in the table below.

	Slope of the Zip Line	Function Rule	Graph
Phase #1	*0.270*	$y = -1.5x^2 + 2.98$	
Phase #2	*0.270*	$y = -1.5x^2 + 2.67$	
Phase #3	*0.270*	$y = -1.5x^2 + 2.33$	

2. How are the function rules alike? What accounts for these similarities?
Each rule has –1.5 as the coefficient of x^2 *because the constant slope of the zip line causes acceleration to be the same in each case.*

3. How are the function rules different? What accounts for these differences?
The value of c in $y = ax^2 + c$ *changes because the action figure starts at a different distance each time.*

4. Graph all three function rules in the same window. Sketch your graph.



5. How are the graphs alike? What accounts for these similarities?

Sample Observations: They all have the same curve because the action figure accelerates at the same rate each time.

6. How are the graphs different? What accounts for these differences?

They have different y-intercepts because the action figure started a different distance away from the motion detector for each phase.

7. In the context of this experiment what is the meaning of the y-intercept in each of your graphs?

The y-intercept is the distance in meters the action figure was from the motion detector when it started down the zip line.

8. In the context of this experiment what is the meaning of the x-intercept in each of your graphs?

The x-intercept is the time in seconds that it took the action figure to travel from one end of the zip line to the other.

9. In general what is the effect on the value of c in the function rule $y = ax^2 + c$ when the starting point on the zip line is closer to the CBR? Why does this happen?

The value of c decreases because the action figure is traveling less distance.

Down Hill Racing

In the Winter Olympics, a snow skiing race was conducted on a mountainside that has a constant slope. The winner of the race skied straight down the hill after starting from a dead stop. His distance from the finish line for the first 6 seconds was recorded each second and is shown in the table.

Elapsed Time in Seconds	Distance from the Finish Line
0	225
1	221
2	209
3	189
4	161
5	125
6	81

What was the elapsed time when he crossed the finish time?

Answer: 7.5 seconds.

The Rainforest Canopy Tour

Tropical rain forests are dense, lush forests located in the tropical regions of the Earth- Central America, northern South America, central Africa, and southeast Asia. Large, tall trees, growing well over 100 feet tall, form the top layer of the rain forest. Their leaves and branches form what is called the "canopy" of the rain forest, shading the forest floor below.

Some entrepreneurs developed what they call "Canopy Tours." If you go on a Canopy Tour, you will be tied in with a harness then sent at high speeds along wire "zip lines" from treetop to treetop, landing on platforms constructed in the treetops.

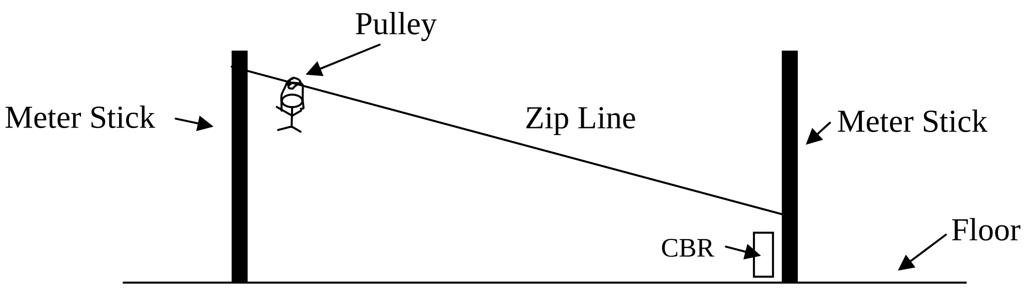
Just how long does it take to travel from one treetop to another? One way to determine the time is to examine the relationship between time and distance traveled.

1. **Imagine stepping off a platform 30 meters above the forest floor and zipping 300 meters along a zip line to the next platform 25 meters above the forest floor. Predict and sketch a distance versus time graph of your journey.**

2. **Suppose, instead of 25 meters, the second platform were 20 meters above the forest floor. How would this change your graph from the graph in question 1? Sketch your new graph.**

When you design a canopy tour, you must predict the time it will take to travel from platform to platform. You can make the predictions using function rules.

To determine these function rules, you will set up an experiment to simulate a person traveling along a zip line from platform to platform high above the floor of the rainforest. You will collect data using a CBR then use your data to determine a function rule.



Set up the experiment as shown so that your zip line is about 3 meters long.

Canopy Tours and Slopes of Zip Lines

To conduct this experiment your group members will need to assume the following roles:

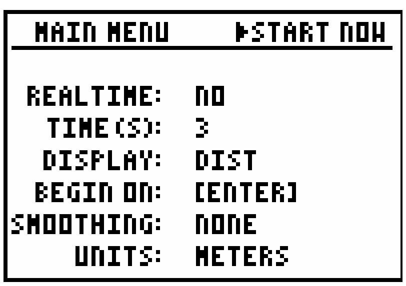
- **2 Meter Stick Managers** pull the zip line tight while keeping the meter stick perpendicular to the floor.
- **Release Manager** holds the pulley on the zip line, against the meter stick. When the CBR operator says “Go,” he/she releases the pulley.
- **CBR Operator** starts data collection when he/she says, “Go.”

Data Collection Phase 1

1. **Place one end of the zip line 20 centimeters above the floor and the other end 100 centimeters above the floor. Pull the zip line tight while keeping the meter sticks perpendicular to the floor. What is the distance between your meter sticks?**

2. **What is the slope of your zip line?**

3. **Collect the data. Sketch the graph from your calculator.**



4. **What type of relationship does your graph appear to represent? How do you know?**

5. **What is the parent function for this type of relationship?**

6. **Graph the parent function over your scatterplot. Sketch your results.**

7. **Is the parent function a good fit to your data? Why or why not?**

8. **Adjust your function rule until you have a curve that fits the data. Write your rule and sketch your graph.**

9. **In phase 2 of this experiment, you will decrease the slope of the zip line. Predict how the decrease in slope will affect your function rule and your graph.**
   Write your predicted function rule and sketch your predicted graph.

Data Collection Phase 2

1. **Place one end of the zip line 20 centimeters above the floor and the other end 85 centimeters above the floor. Pull the zip line tight while keeping the meter sticks perpendicular to the floor. What is the distance between your meter sticks?**

2. **What is the slope of your zip line?**

3. **Collect the data. Sketch the graph from your calculator.**

4. **Determine a function rule that fits your data. Write your rule and sketch your graph.**

5. **In phase 3 of this experiment, you will further decrease the slope of the zip line. Predict how this decrease in slope will affect your function rule and your graph. Write your predicted function rule and sketch your predicted graph.**

Data Collection Phase 3

1. **Place one end of the zip line 20 centimeters above the floor and the other end 70 centimeters above the floor. Pull the zip line tight while keeping the meter sticks perpendicular to the floor. What is the distance between your meter sticks?**

2. **What is the slope of your zip line?**

3. **Collect the data. Sketch the graph from your calculator.**

4. **Determine a function rule that fits your data. Write your rule and sketch your graph.**

Making Connections

1. **Summarize your findings in the table below.**

	Slope of the Zip Line	Function Rule	Graph
Phase #1			
Phase #2			
Phase #3			

2. **How are the function rules alike? What accounts for these similarities?**

3. **How are the function rules different? What accounts for these differences?**

4. **Graph all three function rules in the same window. Sketch your graph.**

5. **How are the graphs alike? What accounts for these similarities?**

6. **How are the graphs different? What accounts for these differences?**

7. **In the context of this experiment, what is the meaning of the y-intercept in each of your graphs?**

8. **In the context of this experiment, what is the meaning of the x-intercept in each of your graphs?**

9. **In general what is the effect on the value of *a* in the function rule $y = ax^2 + c$ when the slope of the zip line decreases? Why does this happen?**

10. **What happens to the function rule as the slope of the zip line approaches 0?**

11. In general what is the effect on the value of *a* in the function rule $y = ax^2 + c$ when the slope of the zip line increases? Why does this happen?

12. What happens to the function rule as the zip line becomes vertical?

13. The Rainforest Canopy Tour Company advertises it longest zip line to be 430 meters from platform to platform. If the slope of this zip line is the same as your slope in phase 2 of this experiment about how much time should it take Jose to zip along the line from one platform to the other?

14. When Jose travels along the 430-meter line, about how far will he be from the next platform after 10.3 seconds?

Canopy Tours and Length of Zip Lines

To conduct this experiment your group members will need to assume the following roles:

- **2 Meter Stick Managers** pull the zip line tight while keeping the meter stick perpendicular to the floor.
- **Release Manager** holds the pulley on the zip line, at different distances from the meter stick. When the CBR operator says "Go," he/she releases the pulley.
- **CBR Operator** starts data collection when he/she says, "Go."

Data Collection Phase 1

1. **Place one end of the zip line 20 centimeters above the floor and the other end 100 centimeters above the floor. Pull the zip line tight while keeping the meter sticks perpendicular to the floor. What is the distance between your meter sticks?**

2. **What is the slope of your zip line?**

3. **Collect the data. For this phase of the data collection the Release Manager needs to start by holding the pulley against the meter stick and releasing it from that position. Sketch the graph from your calculator.**

4. **What type of relationship does your graph appear to represent? How do you know?**

5. **What is the parent function for this type of relationship?**

6. Graph the parent function over your scatterplot. Sketch your results.

7. Is the parent function a good fit to your data? Why or why not?

8. Adjust your function rule until you have a curve that fits the data. Write your rule and sketch your graph.

9. In phase 2 of this experiment, you will decrease the distance your action figure travels on the zip line. Predict how the decrease in distance will affect your function rule and your graph.

Write your predicted function rule and sketch your predicted graph.

Data Collection Phase 2

1. **Once again, place one end of the zip line 20 centimeters above the floor and the other end 100 centimeters above the floor. Pull the zip line tight while keeping the meter sticks perpendicular to the floor. What is the distance between your meter sticks?**

2. **What is the slope of your zip line?**

3. **Collect the data. For this phase of the data collection the Release Manager needs to start by holding the pulley one foot from the meter stick and releasing it from that position. Sketch the graph from your calculator.**

4. **Determine a function rule that fits your data. Write your rule and sketch your graph.**

5. **In phase 3 of this experiment, you will further decrease the distance your action figure travels on the zip line. Predict how the decrease in distance will affect your function rule and your graph.**

   Write your predicted function rule and sketch your predicted graph.

Data Collection Phase 3

1. **Once again, place one end of the zip line 20 centimeters above the floor and the other end 100 centimeters above the floor. Pull the zip line tight while keeping the meter sticks perpendicular to the floor. What is the distance between your meter sticks?**

2. **What is the slope of your zip line?**

3. **Collect the data. For this phase of the data collection the Release Manager needs to start by holding the pulley two feet from the meter stick and releasing it from that position. Sketch the graph from your calculator.**

4. **Determine a function rule that fits your data. Write your rule and sketch your graph.**

Making Connections

5. Summarize your findings in the table below.

	Slope of the Zip Line	Function Rule	Graph
Phase #1			
Phase #2			
Phase #3			

6. How are the function rules alike? What accounts for these similarities?

7. How are the function rules different? What accounts for these differences?

8. Graph all three function rules in the same window. Sketch your graph.

9. How are the graphs alike? What accounts for these similarities?

10. How are the graphs different? What accounts for these differences?

11. In the context of this experiment what is the meaning of the y-intercept in each of your graphs?

12. In the context of this experiment what is the meaning of the x-intercept in each of your graphs?

13. In general what is the effect on the value of c in the function rule $y = ax^2 + c$ when the starting point on the zip line is closer to the CBR? Why does this happen?

Down Hill Racing

In the Winter Olympics, a snow skiing race was conducted on a mountainside that has a constant slope. The winner of the race skied straight down the hill after starting from a dead stop. His distance from the finish line for the first 6 seconds was recorded each second and is shown in the table.

Elapsed Time in Seconds	Distance from the Finish Line
0	225
1	221
2	209
3	189
4	161
5	125
6	81

What was the elapsed time when he crossed the finish time?

1 How does the graph of the function $y = 3.2x^2 + 4.5$ compare to the graph of the function $y = x^2$?

A The graph of $y = 3.2x^2 + 4.5$ opens opposite of $y = x^2$.

B The graph of $y = 3.2x^2 + 4.5$ is a vertical shift of $y = x^2$.

C The graph of $y = 3.2x^2 + 4.5$ is "flatter" than $y = x^2$.

D The graph of $y = 3.2x^2 + 4.5$ is a horizontal shift of $y = x^2$.

2 Frank used a CBR to collect distance versus time data while he rolled a ball down a ramp. He used his data to determine the function rule $y = -8.5x^2 + 13.3$.

Which of the following is a correct assumption about his experiment?

A The ball rolled away from the CBR.

B The ball started 8.5 feet away from the CBR.

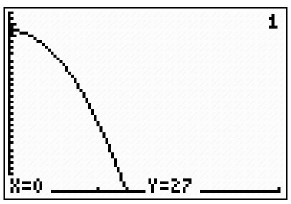
C The ball rolled toward the CBR.

D The ball rolled for 13.3 seconds.

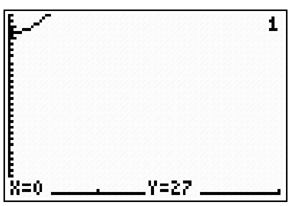
3 Lindsay dropped a book from her second story bedroom window. She determined the function that modeled the distance versus time relationship to be $y = -16x^2 + 18$.

Which of the following graphs could represent dropping a book from a third story window?

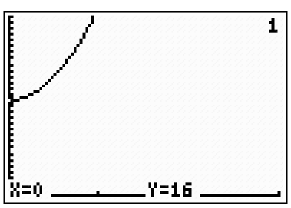
A



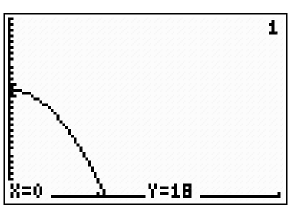
B



C



D



4 Gander collected the data shown in the table.

x	y
0	9
0.2	8.8
0.4	8.3
0.6	7.3
0.8	6.1
1.0	4.4
1.2	2.4

Which function rule best describes this relationship?

A $y = -16x^2 + 9$

B $y = -4.6x^2 + 9$

C $y = -4.9x^2 + 9$

D $y = -9.8x^2 + 9$